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Verification and refinement of spectral magnitude calibrating functions for P-waves

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Abstract

Calibrating functions for determination of P-wave spectral magnitudes calculated by Duda and Yanovskaya (1994) on the basis of the IASP91 velocity model and the PREM Q-model are verified empirically. For this purpose IRIS broadband records for 120 earthquakes are used, each earthquake having been recorded at about 100 stations. The discrepancies indicate that anelastic absorption assumed in PREM is too high.

New calibrating functions are calculated on the basis of the AK135 velocity and anelasticity models (Kennet et al., 1995; Montagner and Kennet, 1996), in which Q in the mantle is higher than in PREM. The verification of the new calibrating functions based on the same observations yields magnitude figures less depending on the epicentral distance. In addition the parameter $\tau_2$in the Liu-Anderson Q(T) model is estimated, proceeding from the assumption that on the average the radiated spectra comply with the ω^2-model. The value of τ_2 was assumed to be 0.1 s in the analysis and its use resulted in the verification of the ω^2-model for the source spectrum.

Introduction

Magnitude calibrating functions of Gutenberg and Richter (1956) developed for body-wave magnitude determinations from narrowband seismograms continue to be widely used in seismological practice. However, the introduction of broadband instruments into seismological observations during the last two decades requires an updating of the magnitude concept and of the magnitude determination procedure. Indeed, it now becomes nearly impossible to estimate the 'amplitude'/'period' ratio, in view of the fact that broadband instruments record ground velocity in the frequency range of as many as 10 octaves, rendering signals more complex than the ones from narrowband instruments. The concept of spectral body wave magnitudes, which depend on period, allows one to avoid the above difficulty, as well as magnitude saturation which arises for large earthquakes.

Attempts to extend the conventional definition of magnitude to the range of periods measured by broadband seismographs have been undertaken by different authors (e.g. Howell et al., 1970; Grant and Mansinha, 1977; Shapira and Kulhanek, 1978; Nortmann and Duda, 1983a; Yacoub, 1983). Such extension requires new calibrating functions which would compensate the loss of energy due to wave propagation, which is obviously different at different periods, the difference being more prominent at short periods. Calibrating functions which depend on the period, in addition to the distance and the source depth, are also necessary for the determination of moment-rate spectra, which are used as the basis for inferences on source processes.

The calibrating functions for spectral magnitudes cannot be obtained on purely empirical grounds, as the procedure would require the knowledge (or joint estimation) of the source spectra. On the other side theoretical calculations, based on a standard velocity and anelasticity Earth's model (Nortmann and Duda, 1983b; Duda and Yanovskaya, 1994) require an empirical verification, in view of the uncertainty of the underlying Earth models, especially of the anelasticity model (Pachova and Yanovskaya, 1992; Duda and Yanovskaya, 1993). Therefore some conceptual

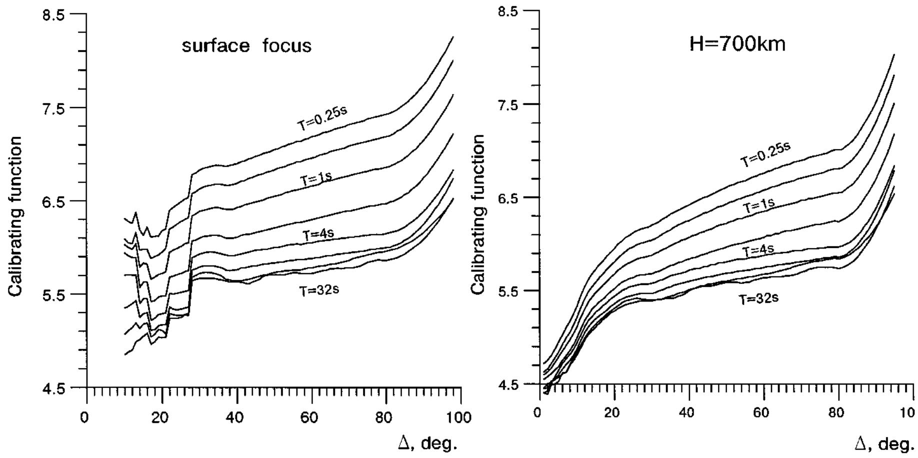

Figure 1. Spectral magnitude calibrating functions of Duda and Yanovskaya (1994) for Z-component of P-wave for surface focus and for the source depth of 700 km.

assumptions about the source spectra need to be introduced, as will be shown below.

Spectral calibrating functions for the periods 0.25–32 s in steps of one octave were calculated by Duda and Yanovskaya (1994) on the basis of the IASP91 velocity model (Kennet and Engdahl, 1991), PREM $Q_m(r)$-model for the absorption band (Dziewonski and Anderson, 1981), assuming a dependence of Q on the period at short periods according to the Liu & Anderson model (Liu et al., 1976):

$$Q(T) = Q_m \frac{2}{\pi} \tan^{-1}\left(\frac{T}{2\pi\tau_2}\right)$$

with $\tau_2 = 0.1$ s. As examples, calibrating functions for the vertical component of the P-wave for a surface focus and for the source depth of 700 km are shown in Figure 1. The computer program PASTA incorporating these calibrating functions for the calculation of the spectral magnitudes as well as of the moment rate spectrum from broadband records was compiled by Roslov (1994). However a preliminary verification of the spectral calibrating functions on the basis of IRIS records for 38 events carried out by Yanovskaya et al. (1996) discovered however some discrepancies, particularly prominent at short periods.

It appeared that these discrepancies require a refinement of the structural models and subsequently of the spectral calibrating functions. In the present investigation the number of events analyzed was increased to 120, in order to substantiate the discrepancies, i.e., the dependence of spectral magnitudes on the epicentral distance for magnitudes calculated on the basis of calibrating functions by Duda & Yanovskaya (1994). Subsequently adjustments to the structural models were made and new calibrating functions computed. Finally, the verification procedure was repeated.

Method for empirical verification of the spectral calibrating functions

The empirical verification of the theoretical spectral calibrating functions (Yanovskaya et al., 1996) is based on the following principle: if the calibrating functions are incorrect, then the estimates of the spectral magnitudes for each event would differ at the stations located at different epicentral distances. Though some other factors affect the differences between the station spectral magnitudes, such as radiation pattern, lateral heterogeneities of the Earth, structure of the crust beneath the stations, etc., they can be suppressed by the statistical approach, employing data for many events in different regions of the Earth. Therefore for each event (j) spectral magnitudes are estimated at the stations (i) located at different epicentral distances Δ_{ij} (so-called *station* magnitudes

$$M_{\text{av}}^j(T) = \frac{1}{N_j} \sum_{i=1}^{N_j} M_i^j(T),$$

where N_j is the number of station estimates. The station residuals $\delta M_i^j(T) = M_i^j(T) - M_{\text{av}}^j(T)$ are then obtained for each event and a statistical analysis is applied to the residuals, which are treated as a func-

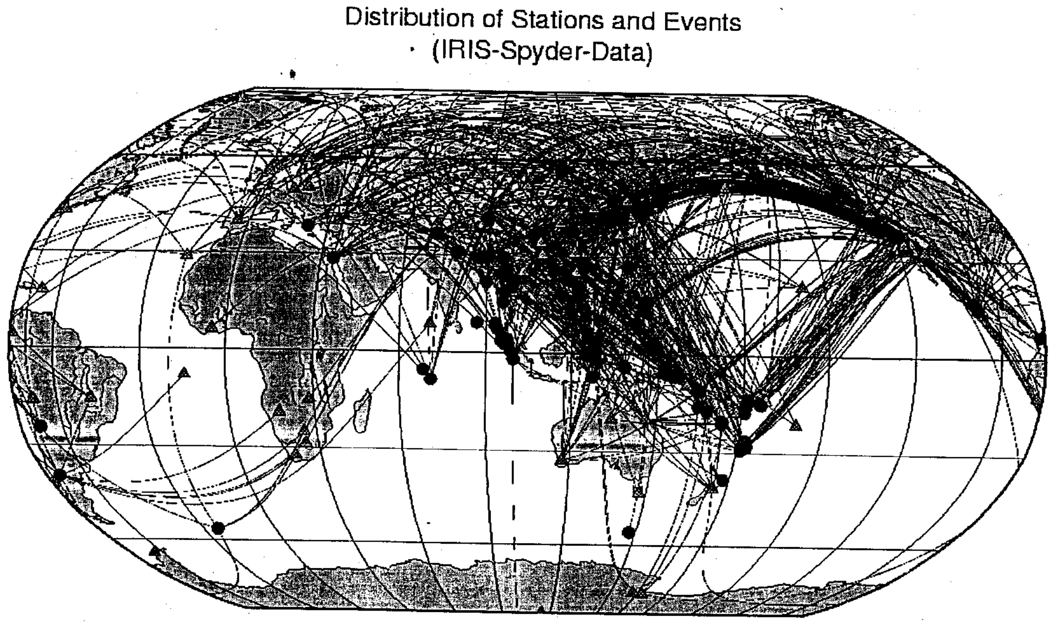

Figure 2. Location of earthquakes, IRIS stations, and the wave paths corresponding to the data used in this study for verification of spectral magnitude calibrating functions.

tion of epicentral distances Δ_{ij} for each period *T*. The residuals are also averaged using a sliding interval of Δ, creating an average function $\langle \delta M_{\mathrm{T}}(\Delta) \rangle$. If the curves $\langle \delta M_{\mathrm{T}}(\Delta) \rangle$ differ significantly from zero, the calibrating functions should be corrected.

Evidently, such approach requires data for many events distributed more or less uniformly over the Earth's surface, recorded at the epicentral distances covering also uniformly the whole range of epicentral distances (up to $\sim 100^{\circ}$). Since a dependence of the residuals on distance is in general different for different focal depths, the analysis could be performed for different focal depths. However, in view of an insufficient number of deeper events, only crustal sources were finally used.

It should be pointed out that the method described above allows to verify only the shape (notably the slope) of each one of the calibrating curves as a function of epicentral distance, but not the difference between the curves for various periods. (This concerns mostly short periods, at which the effect of anelasticity is prominent). However, the difference and the slope are related since they both depend on *Q*: the lower the average *Q*-value in the mantle, the larger both the slope of the curves for high frequencies and the shift between the curves corresponding to different periods.

Data

Data from earthquakes recorded by IRIS stations in the time 1993–1994 are used in the analysis. Only events recorded at $N \geq 5$ stations with sufficiently large signal/noise ratio are taken into consideration. Thus the data set contains only sufficiently strong earthquakes (with m_{b} mainly about 6). The total number of the selected events is equal to 120. The distribution of the epicenters, stations and the wave paths are shown in Figure 2.

Since the majority of the strong earthquakes is concentrated along the Pacific belt, and the IRIS stations are located mostly in Europe and in North America, the number of large epicentral distances is expected to dominate. However, the rejection of records with low signal/noise ratio, which occur more frequently at large distances, resulted in a data set that covers the distance interval 20–90° approximately uniformly (Figure 3).

Results of the analysis

The curves $\langle \delta M_{\mathrm{T}}(\Delta) \rangle$ are calculated by averaging the magnitude residuals δM_i^j in sliding intervals of epicentral distance ranges of 10°, in steps of 5°. The

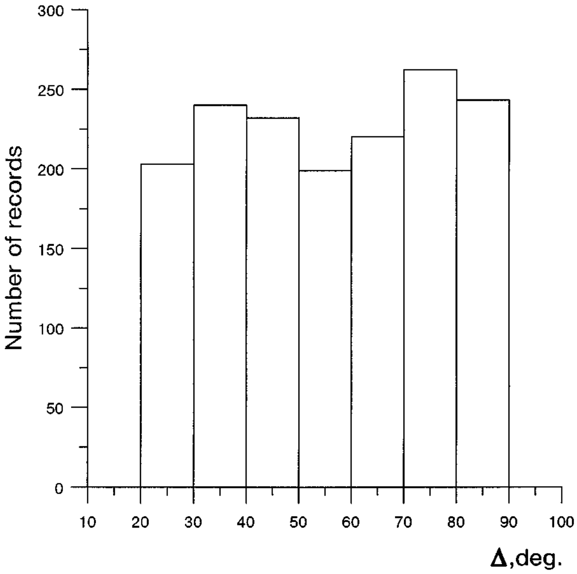

Figure 3. Distribution of the data with epicentral distance. The distribution is approximately uniform in the distance range 20–90°.

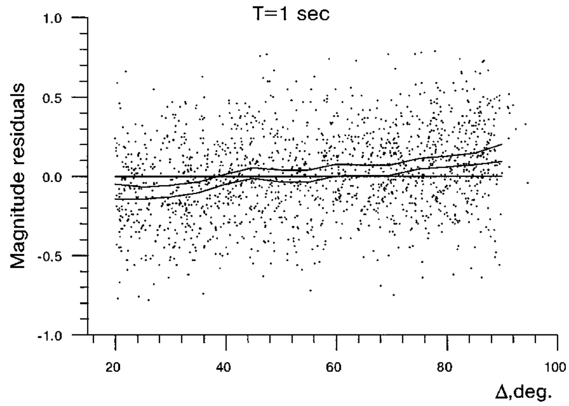

Figure 4. The example of the magnitude residuals δM_i^j displayed as a function of epicentral distance for the period $T = 1$ s. The solid lines bound the 95% confidence band for the curve $\langle \delta M_{T=1}(\Delta) \rangle$ obtained by averaging the data in the sliding interval of epicentral distances 10° with a step 5°.

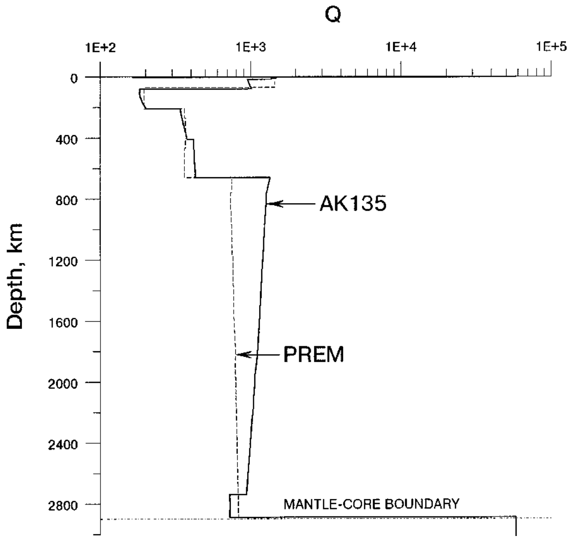

Figure 5. Q_m(r) for PREM model (dashed line) and for AK135 model (solid line) used for calculation of preliminary calibrating functions (Duda and Yanovskaya, 1994) and of the functions in this study respectively.

95% confidence band for the average is determined as $\langle \delta M_T(\Delta) \rangle \pm 1.96\sigma(\Delta)/\sqrt{n(\Delta)}$, where $\sigma(\Delta)$ is standard error of individual measurement, and $n(\Delta)$ is the number of data within the interval $(\Delta - 5^\circ, \Delta + 5^\circ)$.

As an example, $\delta M_i^j(\Delta)$ for the period $T = 1$ s is shown in Figure 4, together with 95% confidence band. It is seen that the curve $\langle \delta M_{T=1}(\Delta) \rangle$ differs significantly from zero, being negative at short and positive at larger distances. Only in the narrow interval of 40°–60° is the residual equal to zero. The curves $\langle \delta M_T(\Delta) \rangle$ for other periods (Figure 7a) display a similar tendency, notably for short periods. Only for the long period $T = 16$ s the slope becomes negative at the distances 20–40°, but here the verification is less reliable, due to possible theoretical limitations.

It should be noted that positive average residuals in the interval 80–90° result from inapplicability of the ray theory for theoretical calculations of the calibrating functions. In this interval the calibrating functions should be determined by extrapolation so as to fit the observations (compare Jansky, Kvasnicka and Duda, 1997).

Nevertheless, even if this interval is eliminated from the analysis, the total trend of the curves $\langle \delta M_T(\Delta) \rangle$ is preserved. The more prominent discrepancy in slope at short periods means that the anelastic absorption is not so strong as predicted by the *Q*-model assumed for calculations. This may be the result of the two alternatives: either $Q_m(r)$ is assumed to be too low, or the high-frequency boundary of the absorption band is shifted to high frequencies, which is equivalent to a too low value of the parameter τ_2 in the Liu-Anderson *Q*(*T*)-model. However, the latter alternative is unlikely: the latest studies on the *Q*(*T*) at high frequencies indicate that the τ_2-value should be of about or even less than 0.1 s (Nortmann and Duda, 1983b; Bock et al., 1982; Bache et al., 1985, 1986; Douglas, 1992). Therefore the more probable conclusion is that $Q_m(r)$ in the real Earth should be higher than in the PREM-model.

Model AK135 as the basis for theoretical spectral calibrating functions

Recently a new velocity and anelasticity model, AK135, was constructed by Kennet et al. (1995), and by Montagner and Kennet (1996) from the data on body wave travel times and on normal modes. According to this model $Q_m(r)$ in the Earth's mantle is indeed higher than in PREM. Figure 5 demonstrates the difference between the two models. The velocity model AK135 also differs slightly from IASP91 but this difference would not affect much the calibrating functions, as was shown by Duda and Yanovskaya (1993). Nevertheless we used both the velocity and Q model AK135 for calculation of the new calibrating functions. The $Q(T)$-model was taken the same as before, i.e. the Liu-Anderson model with $\tau_2 = 0.1$ s.

The calibrating functions for a shallow focus calculated on the basis of the new model are shown together with the old ones in Figure 6. It is seen that at short periods the slope of the new curves is smaller, as is also the shift between the curves for different periods. In the interval 80–90° the functions are obtained by extrapolation so as to fit the observational data.

The spectral magnitudes $M_i^j(T)$ for the same data set and the magnitude residuals δM_i^j were recalculated using the new calibrating functions. Figure 7b shows the confidence limits for the averaged magnitude residuals for $T = 0.25-16$ s (compare with Figure 7a, where the same is shown for the case, when the old calibrating functions were used for determination of the spectral magnitudes). It is clear that Q_{AK135} fits the P-wave amplitude spectra much better than Q_{PREM}.

The value of $\tau_2 = 0.1$ s assumed for calculation of the old calibrating functions was selected so as to comply on the average with the ω^2-model for source spectra (Yanovskaya and Duda, 1994). As noted above, this value of τ_2 is in good agreement with different independent estimates. However, after the correction of the Q_m-model it is necessary to verify the τ_2-value. As before, we assume that on the average, the radiated spectra at the periods shorter than the corner period comply with the ω^2-model. This allows to check the consistency of the shift between the calibrating functions for different periods.

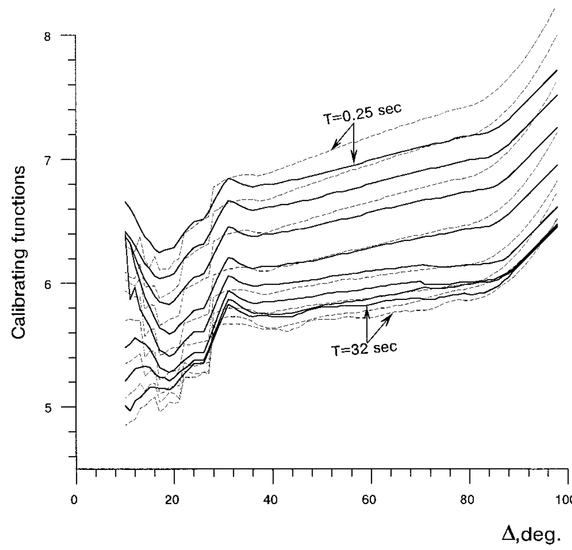

Figure 6. The comparison of the preliminary spectral calibrating functions (dashed line) and spectral calibrating functions on the basis of AK135 model (solid line).

On the basis of the new Q_m-model and three alternative τ_2-values: 0.05 s, 0.1 s and 0.2 s we calculated spectral calibrating functions. Using these calibrating functions we recalculated the spectral magnitudes as averaged from the station spectral magnitudes for 61 of 120 events characterized by the corner period $T_c > 4$ s (Figure 8). For such events at least four spectral magnitude values (for $T = 0.25$, 0.5, 1 and 2 s) correspond to the high frequency part of the source spectrum. These values of the spectral magnitudes were reduced to the same level for all events and approximated by a straight line. The slope of the line together with the shifts between the lines for individual events were estimated by the least-squares' method. The estimates of the slope are listed in the Table below for the selected τ_2-values:

τ_2-values	average slope
0.05	0.41
0.1	1.04
0.2	1.48

Since the spectral magnitude estimates the ground velocity in the source rather than the displacement, the slope of the approximating straight line $M(T)$ at the periods shorter than the corner period should be equal to 1 in case of the ω^2-model. Indeed, the value $\tau_2 = 0.1$ s satisfies this requirement very well.

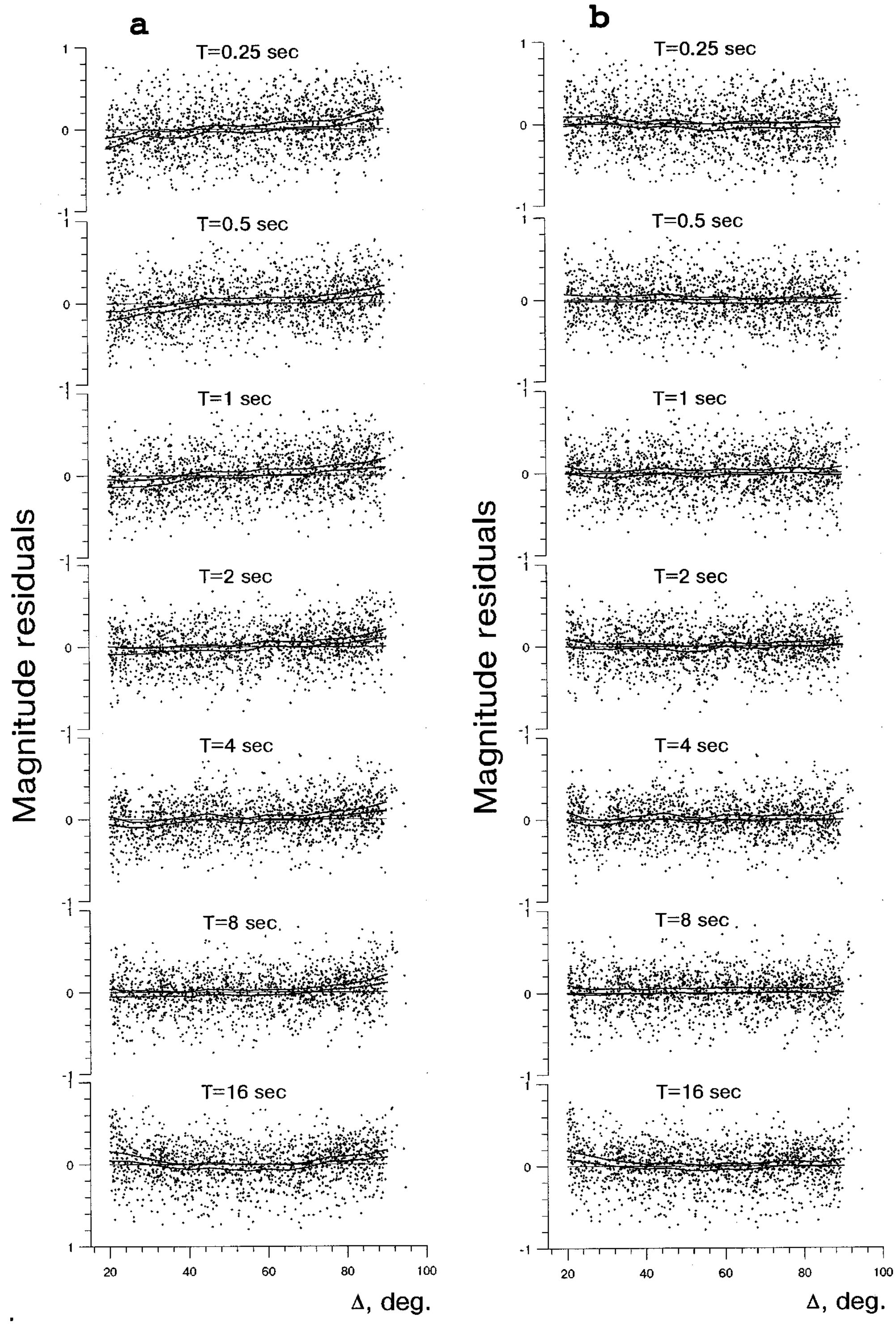

Figure 7. The magnitude residuals and 95% confidence band for the average curves $\left\langle \delta M_T(\Delta) \right\rangle$ for periods $T = 0.25$–16 s calculated using as old calibrating functions (a) as new calibrating functions (b).

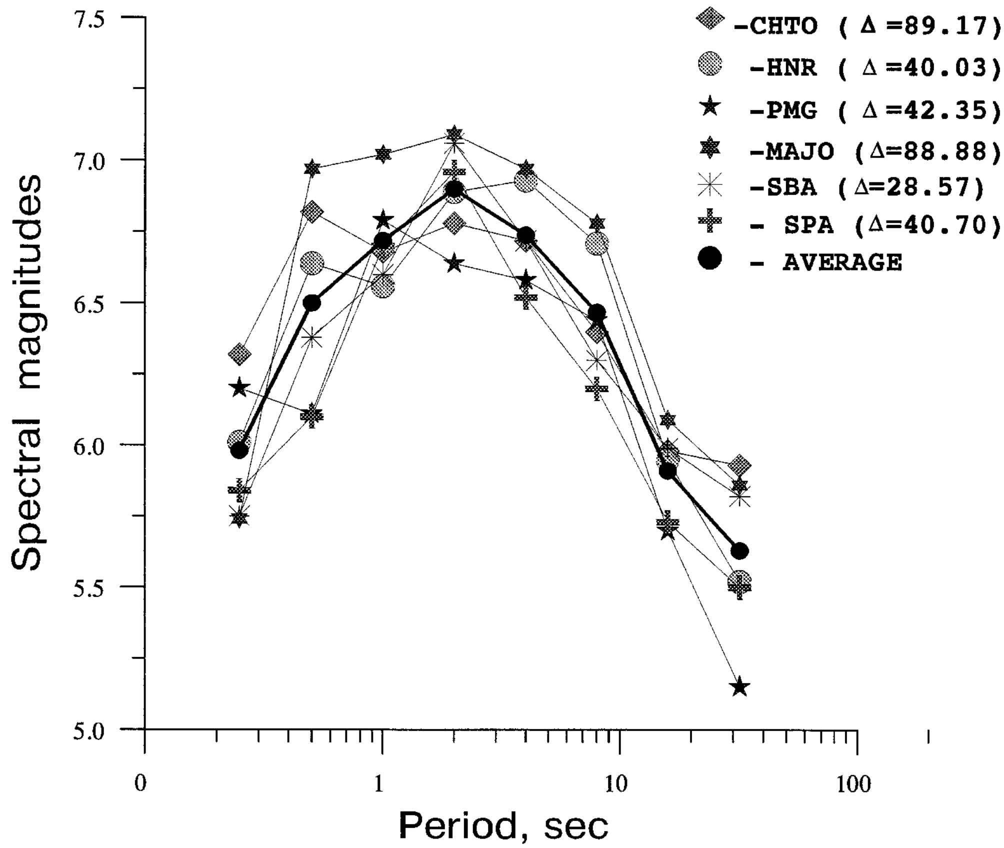

Figure 8. An example of station and average spectral magnitudes of the earthquake on January, 3, 1994 (New Zealand Region) with $m_b = 6.1$ calculated on the basis of new calibrating functions corresponding to $\tau_2 = 0.1$ s. Maximum of the spectral magnitudes corresponds to the corner period $T_c = 2$ s. It is seen that the slope of the curve at periods shorter than the corner period is approximately 1, and at longer periods is about -1, which is in agreement with the ω^2-model.

Conclusions

Empirical verification of the calibrating functions for the determination of spectral magnitudes on the material of broadband IRIS records proves the $Q_m(r)$ in AK135 model to be in better agreement with P-wave amplitudes than that in PREM. Estimation of the τ_2-value, based on the assumption that on the average the radiated spectra comply with the ω^2-model, leads to the value of 0.1 s.

On the basis of the AK135 model and the estimated value of τ_2 the calibrating functions for determination of the spectral magnitudes are calculated.

The functions in table form for periods $T = 0.25$ - 32 s, $h = 0(50)700$ km, $\Delta = 0(1)98^\circ$ (for surface focus $\Delta = 20(1)98^\circ$) are available on request by http://snoopy.phys.spbu.ru/∼lyskova/pasta.html.

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