Traveltime computation for 3D anisotropic media by a finite-difference perturbation method

S.M. Soukina,* D. Gajewski and B.M. Kashtan

Institute of Geophysics, University of Hamburg, Bundesstrasse 55, 20146 Hamburg, Germany

ABSTRACT

The first-order perturbation theory is used for fast 3D computation of quasi-compressional (qP)-wave traveltimes in arbitrarily anisotropic media. For efficiency we implement the perturbation approach using a finite-difference (FD) eikonal solver. Traveltimes in the unperturbed reference medium are computed with an FD eikonal solver, while perturbed traveltimes are obtained by adding a traveltime correction to the traveltimes of the reference medium. The traveltime correction must be computed along the raypath in the reference medium. Since the raypath is not determined in FD eikonal solvers, we approximate rays by linear segments corresponding to the direction of the phase normal of plane wavefronts in each cell. An isotropic medium as a reference medium works well for weak anisotropy. Using a medium with ellipsoidal anisotropy as a background medium in the perturbation approach allows us to consider stronger anisotropy without losing computational speed. The traveltime computation in media with ellipsoidal anisotropy using an FD eikonal solver is fast and accurate. The relative error is below 0.5% for the models investigated in this study. Numerical examples show that the reference model with ellipsoidal anisotropy allows us to compute the traveltime for models with strong anisotropy with an improved accuracy compared with the isotropic reference medium.

INTRODUCTION

Robust and efficient methods for traveltime computation are important in many seismic modelling and inversion applications. There are two major approaches that can be used for computing traveltimes: ray-tracing methods and methods that are based on a direct numerical solution of the eikonal equation using finite differences (e.g. Vidale 1988, 1990; Qin *et al.* 1992). Ray-tracing methods are difficult to implement and time consuming when applied to anisotropic media because for each propagation step an eigenvalue problem must be solved. Similarly to the isotropic case, the ray methods in anisotropic media fail in shadow zones or in the vicinity of a caustic. Finite-difference (FD) eikonal solvers were extended to anisotropic media by Dellinger (1991), Eaton (1993) and Lecomte (1993). Dellinger (1991) used the upwind scheme of Van Trier and Symes (1991) for transversely isotropic media. Eaton (1993) applied an expanding-wavefront scheme on a hexagonal grid in 2D anisotropic models. He approximated one component of the slowness vector using FD and found the root of the sixth-order polynomial for the other component numerically. Lecomte (1993) applied a finite-difference calculation of traveltimes for qP-waves using the method of Podvin and Lecomte (1991) for a 2D model with elliptical and orthorhombic symmetry.

The aim of our work is to compute traveltimes of qP-waves in arbitrarily anisotropic media without solving higher-order polynomials numerically. Perturbation techniques are suitable tools for describing wave propagation in complicated media. We propose combining the perturbation method and the FD eikonal solver for the traveltime computation. Traveltime computation by perturbation with FD eikonal solvers

in isotropic and weakly anisotropic media in 2D was considered by Ettrich and Gajewski (1998). Here we examine the FD perturbation method in the 3D case for strongly anisotropic media.

An isotropic medium as a reference or background medium works well for models with weak anisotropy. Using media with ellipsoidal anisotropy as a background medium in the perturbation approach allows us to consider stronger anisotropy without losing computational speed. The traveltime computation in media with ellipsoidal anisotropy is fast and accurate using the FD eikonal solver. A basic routine for the FD eikonal solver for an elliptically anisotropic medium in the 3D case was presented by Ettrich (1998). He tested his routine in a medium with identical velocities along two directions (ellipsoid with rotational symmetry). The algorithm presented here works for ellipsoidal reference media with three different velocities along the principal axes. Therefore it is possible to use a broader class of ellipsoidal media as reference media.

To minimize errors in the perturbation approach, the reference medium should be chosen to be close to the given anisotropic medium. For the construction of the best-fitting ellipsoidal reference medium we use formulae that were derived by Ettrich, Gajewski and Kashtan (2001). They obtained linear relationships for the coefficients of the ellipsoidal medium that depend on the elastic coefficients of the anisotropic medium. In this study it is assumed that the polarization vector coincides with the phase normal vector. Therefore, only qP-wave anisotropy is approximated.

THE FD PERTURBATION METHOD

The concept of the 2D FD perturbation method was proposed in a paper by Ettrich and Gajewski (1998). Here we present the 3D extension of this method and its implementation for the case of arbitrarily anisotropic media (for P-waves). As a reference model we consider either isotropic or ellipsoidal anisotropic background media. The arbitrarily anisotropic model considered for traveltime computation is also called the perturbed model (perturbed with respect to the reference or background model). For the background model as well as for the perturbed model, velocities or elastic parameters are given on the same regular grid. The reference medium is given by the P-wave velocity for the isotropic case or by the 3×3 symmetric matrix R_{ij} for the ellipsoidal anisotropic case. The parameters of the reference medium are determined from the parameters of the considered perturbed, i.e. arbitrarily anisotropic, medium. The perturbed medium is given by the density normalized elastic parameters c_{ijkl}. Traveltimes for the

isotropic or ellipsoidal anisotropic reference model are computed directly using the FD eikonal solver along expanding wavefronts (Qin *et al.* 1992). For the perturbed model we perform the traveltime computation using standard perturbation techniques. The traveltime corrections are computed along the ray segments corresponding to plane waves in each grid cell for every step of the FD scheme.

Reference medium

To minimize errors in the perturbation approach the reference medium should be chosen to be as close as possible to the given anisotropic medium. For the construction of the ellipsoidal reference medium we used formulae for a best-fitting ellipsoidal reference medium, which were derived by Ettrich *et al.* (2001) using concepts similar to those used for the determination of the best-fitting isotropic medium (see Fedorov 1968). By minimizing the norm of differences between the elastic coefficients of the anisotropic and the ellipsoidal anisotropic media, these authors obtained linear relationships for the elastic parameters of the ellipsoidal reference medium, which depend on the elastic parameters of the considered anisotropic medium. In this study it was assumed that the polarization vector of the qP-wave in the background medium coincides with the phase normal vector. Therefore, only qP-wave anisotropy is considered. To construct the isotropic reference model we use formulae for the best-fitting isotropic medium derived by Fedorov (1968).

FD scheme

Traveltimes in the reference medium are computed with an FD eikonal solver along expanding wavefronts (for isotropic media see Qin *et al.* 1992). The FD eikonal solver for the medium with ellipsoidal anisotropy is based on the eikonal equation for this type of medium, i.e.

$$(\mathbf{p}, \mathbf{Rp}) = 1, \tag{1}$$

where $\mathbf{p}$ is the slowness vector, $\mathbf{R}$ is a 3×3 symmetric matrix of parameters of the ellipsoidal reference medium; ( ) denotes scalar product. The components of the slowness vector are the derivatives of traveltimes with respect to coordinates: $p_i = \frac{\partial t}{\partial x_i}, i = 1, 2, 3$. Similarly to the eikonal equation for isotropic media, two components of the slowness vector can be approximated with finite differences using the already timed gridpoints. Then eikonal equation (1) enables the computation of the third component of the slowness vector, if the medium parameters are known. In our algorithm the approximating formulae of the eikonal equation for an ellipsoidal

anisotropic medium are used. These formulae were proposed by Ettrich (1998). They are analogous to Vidale's (1990) approximating formulae of the eikonal equation for 3D isotropic media.

To retain causality and to guarantee stability, we use a scheme of expanding wavefronts (Qin *et al.* 1992). Following this scheme, after the gridpoint closest to the source location is timed (see Vidale 1988, 1990), the point with the minimum traveltime along the outer perimeter of timed points is determined. Because this is the point at which the wavefront first reaches the perimeter surface, the solution region should be expanded from this point first. All untimed points next to the point with the minimum traveltime are timed using the FD approximating formulae. These newly timed points are used to form the update perimeter surface of the timed points. This procedure is repeated until all points inside the model are timed (Qin *et al.* 1992). In isotropic media this procedure provides the causal expansion of the FD scheme. In anisotropic

media, however, two types of velocity need to be considered: the ray velocity and the phase velocity. The propagation of energy and, therefore, the causal expansion of the FD scheme is governed by the ray-velocity vector. For every step of the FD scheme we must compare the direction of the scheme expansion with the direction of the ray velocity. The traveltime at a given gridpoint has been successfully computed if the directions coincide, otherwise the point with minimum traveltime is not a point for a causal expansion and we have to consider the point with the next minimum traveltime. The phase velocity is determined from the eikonal equation, but the ray velocity in ellipsoidal anisotropic media is computed by the expression $\mathbf{V_g = Rp}$.

To investigate the accuracy of the traveltime computation by the 3D FD eikonal solver for an ellipsoidal anisotropic medium, we consider a homogeneous model since analytical solutions for traveltime computations exist. Figure 1 shows the numerically computed wavefronts for two different
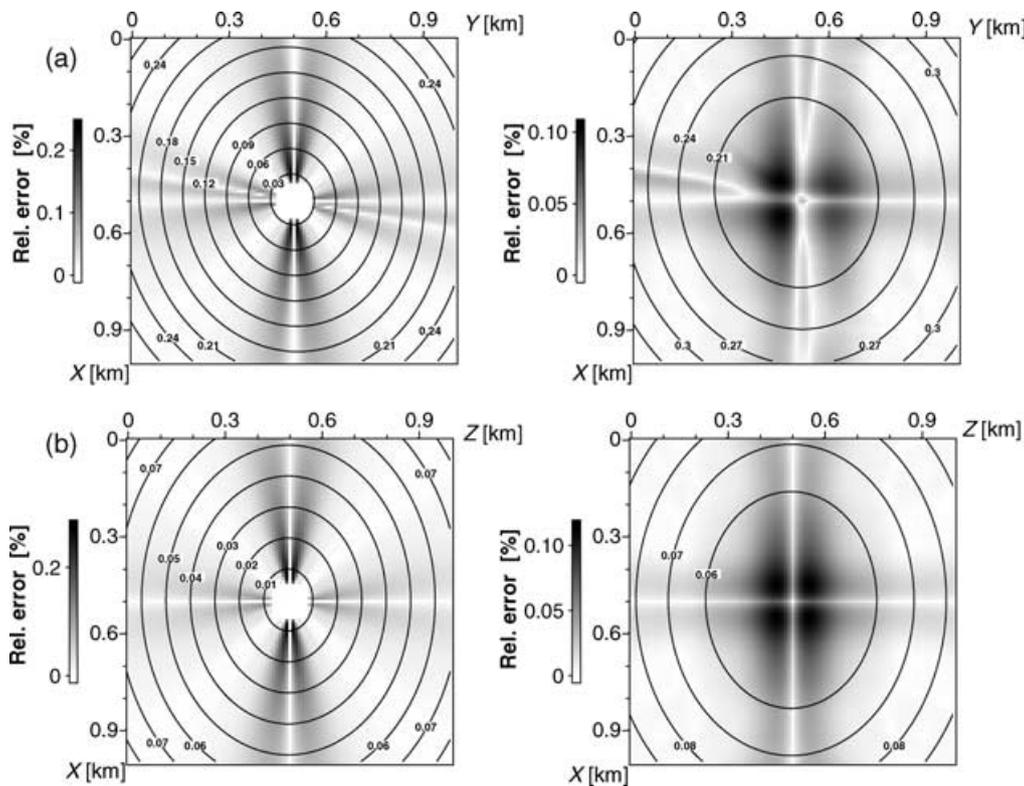
Figure 1 Wavefronts in a homogeneous ellipsoidal anisotropic model: (a) in the elliptical reference medium for triclinic sandstone (for the parameters, see (6)); (b) in the elliptical reference medium for olivine (for the parameters, see (4)). For each model there are two vertical slices with offsets 0 km (left) and 0.4 km (right) from the source, located at point (0.5, 0.5, 0.5) km. The underlying grey-scale images show the relative errors in the traveltime computation with respect to the analytically computed traveltimes. The maximum of the relative errors in the whole 3D model does not exceed 0.35% in case (a) and 0.5% in case (b).

ellipsoidal anisotropic models. These models correspond to the best-fitting ellipsoidal reference models of triclinic sandstone (Fig. 1a) and orthorhombic olivine (Fig. 1b) (for the elastic parameters of the ellipsoidal media used, see equations (4) and (6)). Because it is a 3D model, two vertical slices are displayed: one slice includes the source position (left-hand side of Fig. 1), the other slice has a lateral offset from the source (right-hand side of Fig. 1). The underlying grey-scale images show the relative errors with respect to analytically computed traveltimes. The numerical examples demonstrate that the traveltimes in elliptical anisotropic media are computed with high accuracy using the FD eikonal solver described above. The maximum error for the examples shown is below 0.5%. This error is attributed entirely to the fact that the eikonal equation for ellipsoidal anisotropic media is expressed by finite differences. It is similar to the well-known errors obtained by expressing the eikonal equation for isotropic media by finite differences.

The computation of traveltimes in ellipsoidal anisotropic media is the first step in the approach to computing traveltimes in arbitrarily anisotropic media. Media with ellipsoidal anisotropy will serve as background models in the perturbation approach to computing traveltimes in media with strong arbitrary anisotropy. The perturbation scheme is discussed in the next section.

Basic perturbation formulae

To compute traveltimes for an arbitrarily anisotropic medium, a perturbation scheme is embedded in the FD eikonal solver: traveltimes at every gridpoint of the reference model are computed using an FD eikonal solver for isotropic media (in the case of weak anisotropy) or ellipsoidal anisotropic media (in the case of strong anisotropy). The perturbation method is used for the computation of the traveltime corrections to yield traveltimes in the perturbed (arbitrarily anisotropic) model.

To compute the traveltime correction, the raypath between source and receivers in the reference medium must be known. Rays are not determined in the FD method. In the FD method, traveltimes are computed on a discrete grid assuming local plane wavefronts inside the grid cell. Therefore, we compute the traveltime correction along the ray segments corresponding to the plane wavefronts in each cell. The ray segment is a straight line between the gridpoint (see Fig. 2), where the traveltime needs to be computed (point $A7$), and the point where the ray crosses the cell boundary (point N), which is defined by the orientation of the ray-velocity vector. The ray
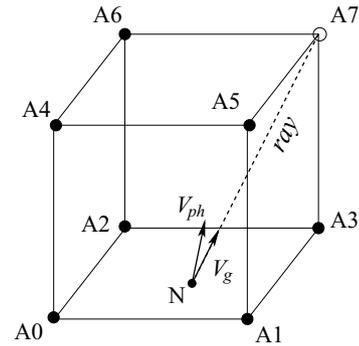
Figure 2 Grid cell with already timed points $A0$–$A6$. A local plane wavefront is assumed to propagate through the cell to compute the traveltime at $A7$. V_{ph} is the phase velocity. The raypath in the cell is defined by the direction of the ray velocity V_g. The length of the ray segment is given by the intersection with the cell boundary at point N and gridpoint $A7$.

velocity for the reference ellipsoidal medium is given by the simple formula: $\mathbf{V_g} = \mathbf{Rp}$. Here, $\mathbf{R}$ is the matrix of the medium parameters (see (1)) and $\mathbf{p}$ is the slowness vector computed for this grid cell using eikonal equation (1). The traveltime $\Delta t'(N)$ at the intersection of the ray segment with the cell boundary (point N, see Fig. 2) is obtained by linear interpolation between the corner points $A0$ to $A3$ of the cell. Therefore, the traveltime at point $A7$ for the perturbed model is given by

$$t_{\text{pert}}(A7) = t_{\text{ref}}(A7) + \Delta t'(N) + \Delta t(N, A7),$$

where t_{pert} is the traveltime in the perturbed model and t_{ref} is the traveltime in the reference model, $\Delta t' = t_{\text{pert}}(N) - t_{\text{ref}}(N)$ is the difference between the traveltimes at point N in the perturbed and reference models, and $\Delta t(N, A7)$ is the traveltime correction, given by

$$\Delta t(N, A7) = -\frac{1}{2} \int_{t_{\text{ref}}(N)}^{t_{\text{ref}}(A7)} \Delta c_{ijkl}\, p_i\, p_l \mathrm{g}_j \mathrm{g}_k \, \mathrm{d}t, \qquad (2)$$

where

$$\Delta c_{ijkl} = c_{ijkl}^{\text{pert}} - c_{ijkl}^{\text{ref}}.$$

The parameters $c_{ijkl}^{(\text{pert})}$ are the density normalized elastic coefficients of the anisotropic medium, p_i are the components of the slowness vector, and g_j are the components of the qP-wave polarization vector. The vectors $\mathbf{p}$ and $\mathbf{g}$ depend on the reference medium. For isotropic reference media, the P-wave polarization is a unit vector which is normal to the wavefront and the density normalized elastic coefficients are defined by

$c_{ijkl}^{\text{ref}} = \frac{\lambda}{\rho}\delta_{ij}\delta_{kl} + \frac{\mu}{\rho}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$. In the case of the ellipsoidal reference medium, we substitute the wavefront normal vector **n** instead of the unknown polarization vector **g** (similar to the approach used by Ettrich *et al.* 2001). Therefore, the traveltime correction reads

$$\Delta t(N, A7) \approx -\frac{1}{2}\int_{t_{\text{ref}}(N)}^{t_{\text{ref}}(A7)}\left(c_{ijkl}^{\text{pert}}n_j\,n_k - R_{il}\right)p_i\,p_l\,\mathrm{d}t, \qquad (3)$$

where R_{il} is a 3×3 matrix of an ellipsoidal reference medium as given in (1). The assumption that the orientation of the polarization vector coincides with the orientation of the wavefront normal introduces an additional error, which is quantified by numerical examples in the following section.

NUMERICAL RESULTS

To illustrate the traveltime computation by the FD perturbation method in 3D anisotropic media, we choose two types of symmetry: triclinic and orthorhombic. To obtain an estimate of the accuracy, we consider homogeneous media first, since exact solutions for traveltimes are available. For comparison, traveltimes for all models are computed for isotropic and ellipsoidal anisotropic reference or background models. In the figures, traveltimes are displayed as first-arrival wavefronts. Traveltimes in the perturbed medium are called exact traveltimes (exact wavefronts) if they are computed directly by anisotropic ray tracing or analytically. They are considered to be exact with respect to the proved accuracy of the method used.

The traveltime errors in the examples are caused by three sources: the error in the numerical computation of traveltimes in a reference ellipsoidal (or isotropic) model using an FD method, which we have already quantified above, the inherent error in the perturbation technique and the error caused by approximating the polarization vector by the wavefront normal vector (in the case of an ellipsoidal reference model). A model cube of 101 gridpoints along each spatial direction is considered. The grid spacing is 10 m. The source is located at the centre of the models, at point (0.5, 0.5, 0.5) km. A cubic region of 11 gridpoints around the source is initialized using the exact solution. Since 3D models are presented, we always show two vertical slices: with zero offset and non-zero offset from the source.

First, we consider a model with orthorhombic anisotropic elastic parameters of olivine (parameters taken from Babuska and Cara 1991). The density normalized parameters (in km^2/s^2) are

$$\begin{pmatrix} 97.77 & 21.62 & 20.05 & 0. & 0. & 0. \\ & 71.0 & 22.83 & 0. & 0. & 0. \\ & & 59.68 & 0. & 0. & 0. \\ & & & 19.51 & 0. & 0. \\ & & & & 23.86 & 0. \\ & & & & & 23.77 \end{pmatrix} \rightarrow$$

$$R_{il} = \begin{pmatrix} 91.99 & 0 & 0 \\ & 67.35 & 0 \\ & & 57.25 \end{pmatrix}; \quad V_p = 8.5\,\text{km/s}. \qquad (4)$$

The coefficients for the best-fitting ellipsoid reference medium (Ettrich *et al.* 2001) and the P-wave velocity for the best-fitting isotropic reference medium (Fedorov 1968) are listed below the density normalized elastic parameters. To compute traveltimes in this medium of strong qP-wave anisotropy we use three types of reference medium: isotropic, ellipsoidal anisotropic and transversely isotropic with elliptical qP-wavefront. The isotropic reference medium is constructed using formulae for the best-fitting isotropic reference medium. In this reference medium, the polarization of the P-wave is a unit vector in the direction of the slowness vectors. Figures 3(a) and 4(a) display the exact wavefronts in the olivine model (solid lines) and the circular wavefronts for the best-fitting isotropic reference medium (dashed lines). Figures 3(b) and 4(b) show the exact wavefronts and the wavefronts constructed from the traveltimes using the FD perturbation method for the best-fitting isotropic background. We observe the largest relative errors between exact (dashed lines) and FD perturbation traveltimes (solid lines) where the isotropic reference medium gives the worst fit. The maximum relative error for the whole 3D model is 3.9%. Such a large error is expected, because the anisotropy of the perturbed model is too strong to use an isotropic reference medium.

The ellipsoidal anisotropic reference medium for the perturbation method allows us to consider stronger anisotropy. However, for this type of reference medium, we approximate the polarization vector by the phase normal vector to compute the traveltime correction (see (2)). The ellipsoidal anisotropic background medium is constructed using formulae for the best-fitting ellipsoidal anisotropic medium (Ettrich *et al.* 2001). We use (3) instead of (2) to compute the traveltime corrections in the perturbed model. Figures 3(c) and 4(c) show the exact wavefronts in olivine (solid lines) and the reference wavefronts in the best-fitting ellipsoidal anisotropic (ellipsoidal) medium (dashed lines). This reference model gives a better fit to the perturbed medium than the isotropic reference model. From Figs 3(d) and 4(d), we
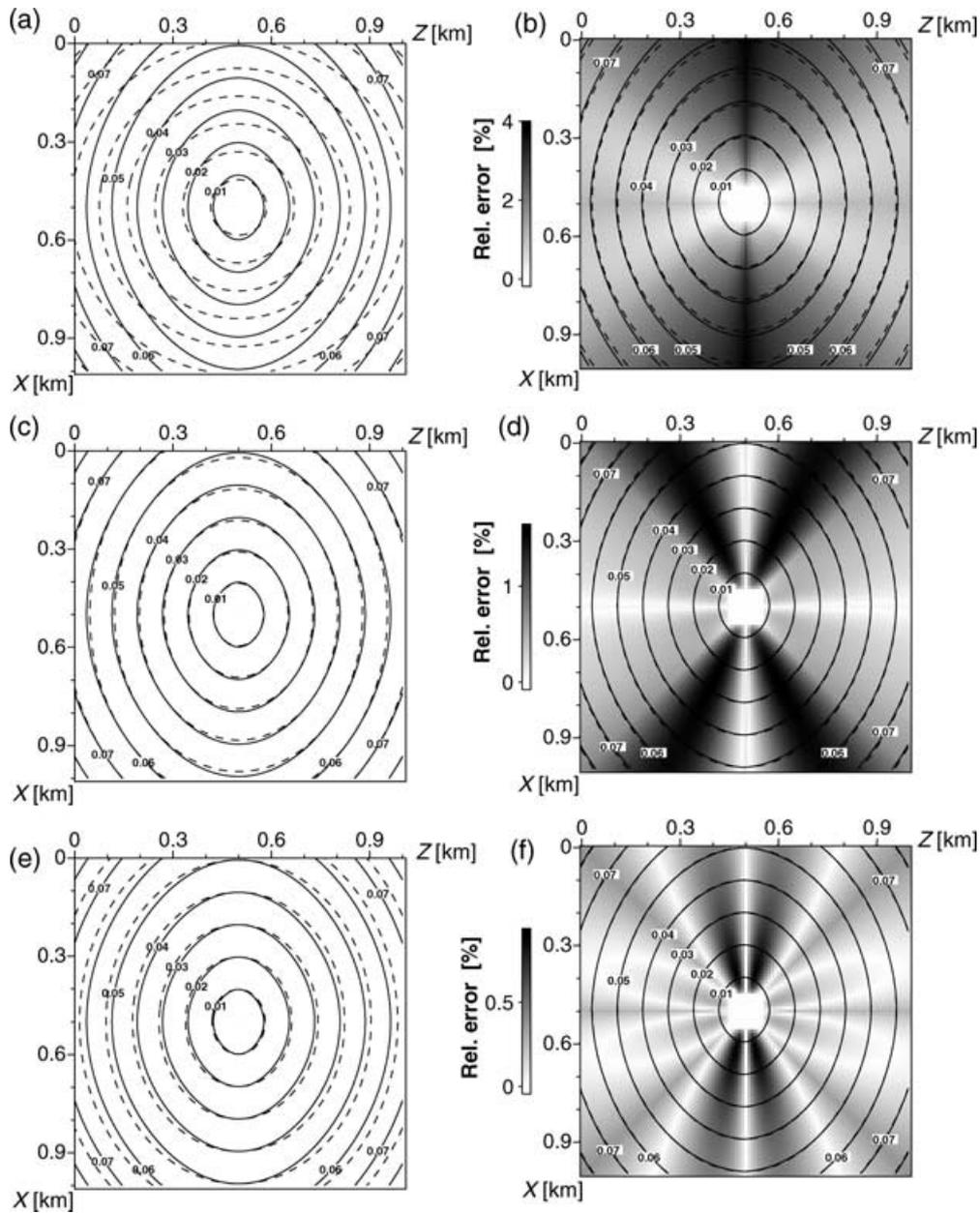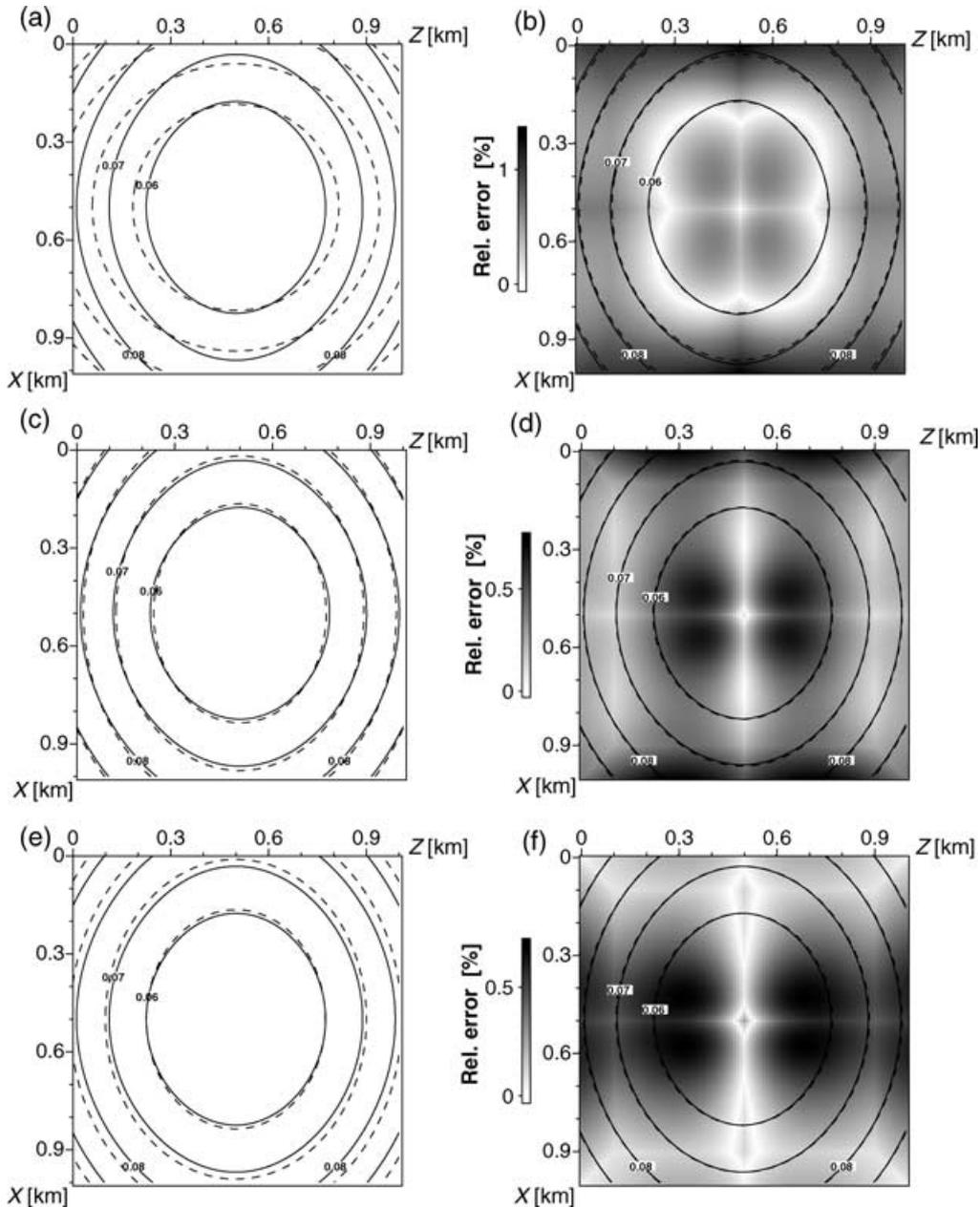
Figure 3 Wavefronts in a homogeneous model of olivine; vertical slice including the source at (0.5, 0.5, 0.5) km. In the left-hand diagrams, solid lines show exact wavefronts and dashed lines show wavefronts in the reference models: (a) isotropic model; (c) ellipsoidal anisotropic model; (e) transversely isotropic model. The diagrams on the right-hand side show the FD perturbation method wavefronts (solid) for the corresponding reference model and exact wavefronts (dashed), with the underlying grey-scale images of relative errors. Note that the error scales are different in (b), (d) and (f).

see that the accuracy of the computation is higher than for the isotropic background media. The maximum relative error for the whole 3D model is 1.89%. This error is mainly due to the approximation of the unknown polarization vector by the phase normal, while the error produced by the trav-

eltime computation using the FD eikonal solver in the reference model does not exceed 0.5% (see discussion above and Fig. 1b).

According to Burridge, Chadwick and Norric (1983), there are three possibilities for simplifying orthorhombic symmetry

Figure 4 Wavefronts in a homogeneous model of olivine; vertical slice with offset 0.4 km from the source at (0.5, 0.5, 0.5) km. In the left-hand diagrams, solid lines show exact wavefronts and dashed lines show wavefronts in reference models: (a) isotropic model; (c) ellipsoidal anisotropic model; (e) transversely isotropic model. The right-hand side diagrams show FD perturbation method wavefronts (solid) for the corresponding reference model and exact wavefronts (dashed), with the underlying grey-scale images of relative errors. Note that the error scales are different in (b), (d) and (f).

to ellipsoidal symmetry. One possibility is a transversely isotropic medium with elliptical qP-wavefronts. In this case we can compute the correct polarization vector for the ellipsoidal anisotropic reference medium. A transversely isotropic medium with axis of symmetry along the X-axis has ellip- soidal symmetry if the non-zero elastic parameters satisfy the relationships:

$$C_{22} = C_{33}; \quad C_{12} = C_{13}; \quad C_{55} = C_{66};$$

$$C_{44} = \frac{1}{2}(C_{22} - C_{23}); \quad C_{55} = \frac{C_{11}C_{22} - C_{12}^2}{C_{11} + C_{22} + 2C_{12}}. \tag{5}$$

Due to the non-linear relationships between the elastic parameters in (5), it is simple to construct the reference medium. The transversely isotropic reference medium for olivine using (5) was constructed as follows:

$$C_{11} = C_{11}^{oliv}; \ C_{12} = \frac{1}{2}\left(C_{12}^{oliv} + C_{13}^{oliv}\right);$$

$$C_{22} = \frac{1}{2}\left(C_{22}^{oliv} + C_{33}^{oliv}\right); \ C_{23} = C_{23}^{oliv}.$$

Figures 3(e) and 4(e) show the exact wavefronts in olivine and the wavefronts in the reference medium with transversely isotropic symmetry using (5). Underlying grey-scale images in Figs 3(f) and 4(f) show the relative errors. The maximum of the relative errors in the whole 3D model does not exceed 1.2%. We observe that the largest relative errors are located where bigger differences in the angle between the wavefront normals in perturbed and reference media occur (in other words, the raypaths are too different). The perturbation method gives high accuracy when the raypaths in both media are close to each other.

In the next numerical example we consider a model of triclinic sandstone (parameters taken from Mensch and Rasolofosaon 1997). The density normalized parameters are (in km^2/s^2)

$$\begin{pmatrix} 6.77 & 0.62 & 1.0 & -0.48 & 0. & -0.24 \\ & 4.95 & 0.43 & 0.38 & 0.67 & 0.52 \\ & & 5.09 & -0.28 & 0.09 & -0.09 \\ & & & 2.35 & 0.09 & 0. \\ & & & & 2.45 & 0.0 \\ & & & & & 2.88 \end{pmatrix} \rightarrow$$

$$R_{il} = \begin{pmatrix} 6.88 & 0.27 & 0.27 \\ & 5.10 & -0.05 \\ & & 5.08 \end{pmatrix}; V_p = 2.38 \, km/s. \qquad (6)$$
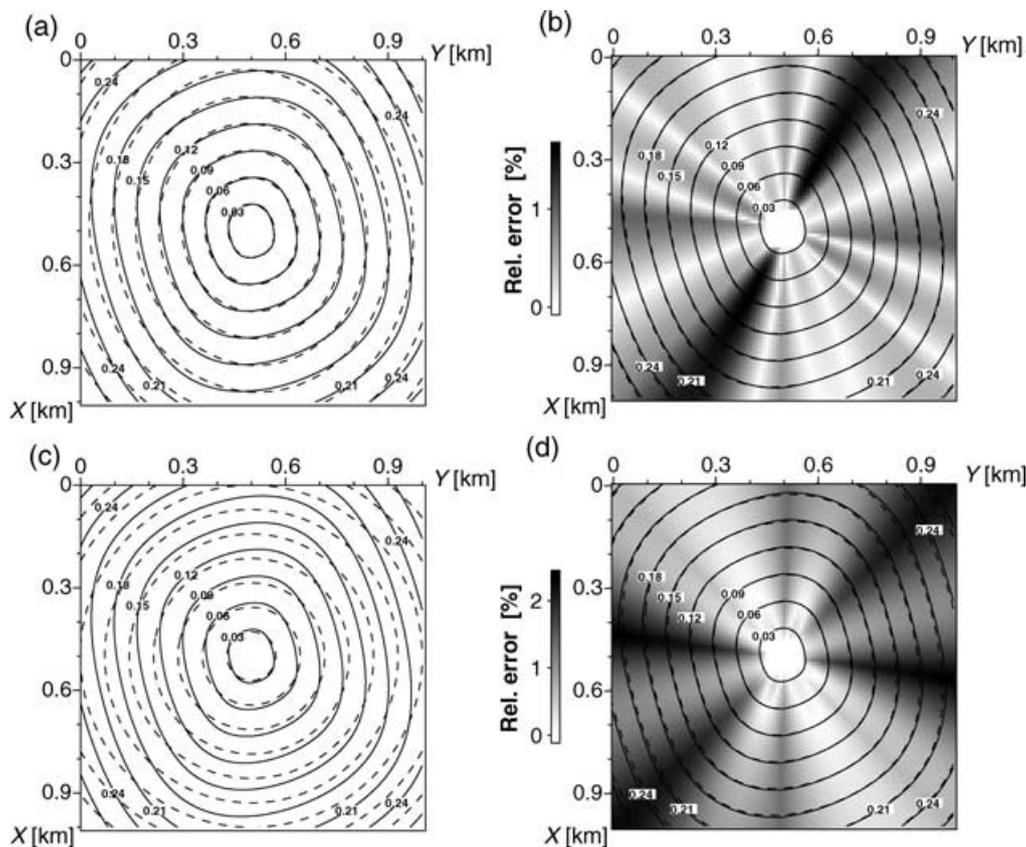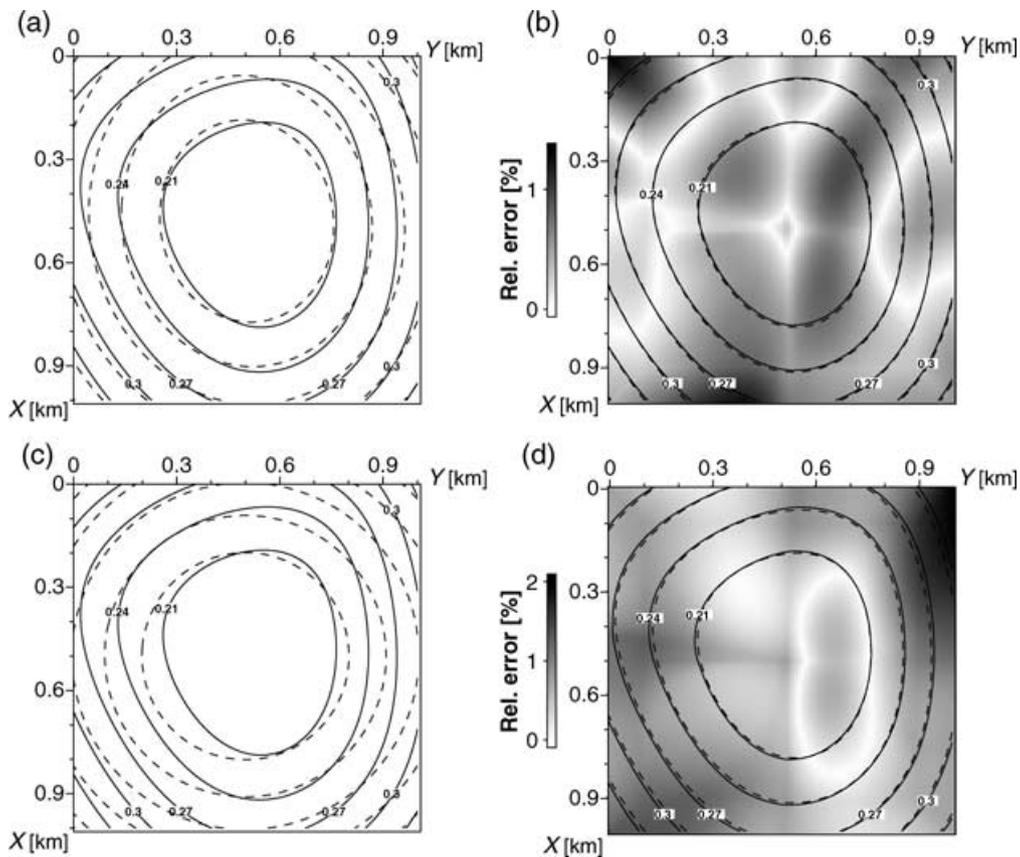

Figure 5 Wavefronts in a homogeneous model of triclinic sandstone; vertical slice including the source at (0.5, 0.5, 0.5) km. The left-hand side diagrams show exact wavefronts (solid) and reference wavefronts (dashed): (a) ellipsoidal anisotropic model; (c) isotropic model. The right-hand side diagrams show FD perturbation method wavefronts (solid) for the corresponding reference model and exact wavefronts (dashed), with the underlying grey-scale images of relative errors: (b) ellipsoidal anisotropic reference model; (d) isotropic reference medium. Note that the error scales are different in (b) and (d).

The coefficients for the best-fitting ellipsoid reference medium and the P-wave velocity for the best-fitting isotropic reference medium are listed below the density normalized elastic parameters. This medium has strong P-wave anisotropy and irregular velocity surfaces caused by the relatively small non-orthorhombic coefficients. The left-hand sides of Figs 5 and 6 show exact wavefronts in the triclinic sandstone (solid lines) and reference wavefronts (dashed lines) in the ellipsoidal model (Figs 5a and 6a) and in the isotropic model (Figs 5c and 6c). The right-hand sides show results of the numerical computation by the FD perturbation method using the ellipsoidal background medium (Figs 5b and 6b) and the background isotropic reference medium (Figs 5d and 6d). The underlying grey-scale images show relative errors with respect to exact solutions. The maximum relative error in the whole 3D model is 2.26% when the ellipsoidal reference medium is used and 2.40% for the isotropic reference medium. Although the maximum error in traveltimes using the ellipsoidal anisotropic reference medium is close to the maximum error in the traveltimes for the isotropic medium for this particular model, we see in Figs 5 and 6 that the accuracy for the ellipsoidal reference medium is considerably better for most parts of the model (mostly light grey despite a different grey scale from that used for the isotropic model).

The numerical examples presented here were homogeneous in order to validate the method and to investigate the errors, since for heterogeneous arbitrarily anisotropic models no exact solutions for computation of traveltimes are available. The FD eikonal solver combined with the perturbation approach can be used to compute traveltimes in heterogeneous anisotropic media without any change to the technique described above. Figure 7 shows the results of the traveltime computation for a heterogeneous model. The heterogeneous model consists of two parts: a triclinic homogeneous layer and

Figure 6 Wavefronts in a homogeneous model of triclinic sandstone; vertical slice with offset 0.4 km from the source at (0.5, 0.5, 0.5) km. The left-hand side diagrams show exact wavefronts (solid) and reference wavefronts (dashed): (a) ellipsoidal anisotropic model; (c) isotropic model. The right-hand side diagrams show FD perturbation method wavefronts (solid) for the corresponding reference model and exact wavefronts (dashed), with the underlying grey-scale images of relative errors: (b) ellipsoidal anisotropic reference model; (d) isotropic reference medium. Note that the error scales are different in (b) and (d).
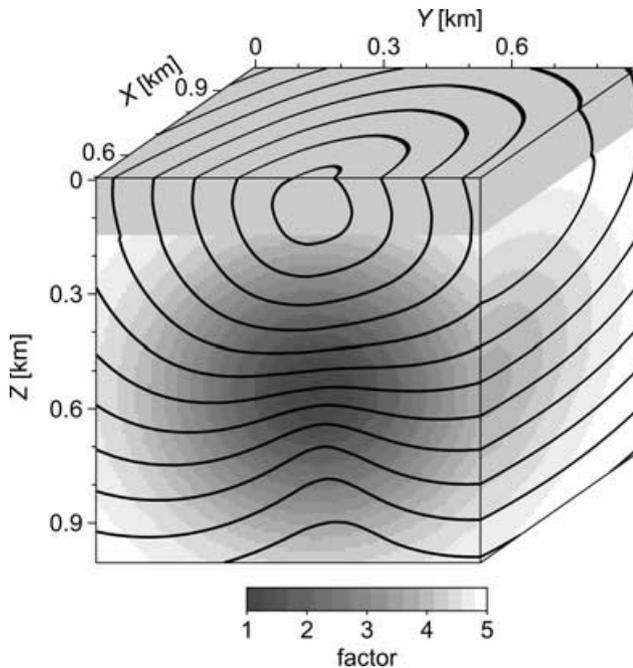
Figure 7 Wavefronts in a heterogeneous triclinic model. The model consists of two parts: a triclinic homogeneous layer and a layer which is constructed by multiplying the elastic parameters of triclinic sandstone in each cell by some factor. The underlying image shows the values of the factor for different cells.

a layer which is constructed by multiplying the elastic parameters of triclinic sandstone in each cell by some factor. The values of the factor correspond to a parabolic lens and vary from 1 to 5. The underlying image in Fig. 7 shows the values of the factor for different cells.

CONCLUSION

We have presented a finite-difference perturbation method for the efficient computation of traveltimes for qP-waves in arbitrarily anisotropic 3D media. An arbitrarily anisotropic medium is considered as a perturbed model with respect to a simple reference medium. Traveltimes in the reference medium are computed using the FD eikonal solver which allows fast and accurate computation of traveltimes. Traveltimes in the perturbed medium are obtained by adding traveltime corrections to the traveltimes of the reference medium using the perturbation approach. Instead of the raypath between source and gridpoint, we use the ray segments corresponding to plane wavefronts in each grid cell.

We propose using models with ellipsoidal symmetry as reference models in order to compute traveltimes for strongly anisotropic media. An ellipsoidal anisotropic medium approximates a strong anisotropic medium better than an isotropic one. The corresponding eikonal equation is only slightly more complex than for the isotropic case. The primary source of errors related to the ellipsoidal anisotropic medium is the unknown polarization vector which is needed to carry out the traveltime correction using the perturbation method. We have substituted the unknown polarization vector by the phase normal vector to compute the traveltime correction and have compared the accuracy of the traveltime computations with the accuracy in the case of an isotropic reference medium. The maximum relative error in the whole 3D model for the olivine model with respect to the isotropic reference model was 3.9%. For the ellipsoidal anisotropic background model it was 1.89%. In the case of the isotropic reference model, the deviation between the perturbed model and the reference model is too large. In this case, the error caused by the perturbation scheme is larger than the error caused by the approximation of the unknown polarization vectors in the case of an ellipsoidal anisotropic reference medium.

The presented FD eikonal perturbation technique can be used for the fast computation of quasi-P-wave traveltimes in arbitrarily anisotropic media. It provides an efficient tool for computing diffraction surfaces for the migration of reflection data from anisotropic media.

ACKNOWLEDGEMENTS

REFERENCES

Babuska V. and Cara M. 1991. Seismic anisotropy in the Earth. In: *Modern Approaches in Geophysics*, Vol. 10. Kluwer Academic Publishers.

Burridge R., Chadwick P. and Norric A.N. 1993. Fundamental elastodynamic solutions for anisotropic media with ellipsoidal slowness surfaces. *Proceedings of the Royal Society of London A* **440**, 655–681.

Dellinger J. 1991. Anisotropic finite-difference traveltimes. 61st SEG Meeting, Houston, USA, Expanded Abstracts, 1530–1533.

Eaton D.W.S. 1993. Finite difference traveltime calculation for anisotropic media. *Geophysical Journal International* **114**, 273–280.

Ettrich N. 1998. FD eikonal solver for 3D anisotropic media. 68th SEG Meeting, New Orleans, USA, Expanded Abstracts, 1957–1960.

Ettrich N. and Gajewski D. 1998. Traveltime computation by perturbation with FD-eikonal solvers in isotropic and weakly anisotropic media. *Geophysics* **63**, 1066–1078.

Ettrich N., Gajewski D. and Kashtan B. 2001. Reference ellipsoids for anisotropic media. *Geophysical Prospecting* **49**, 321–334.

Fedorov F.I. 1968. *Theory of Elastic Waves in Crystals*. Plenum Press, New York.

Jech J. and Pšenčík I. 1989. First order perturbation method for anisotropic media. *Geophysical Journal International* **99**, 369–376.

Lecomte I. 1993. Finite difference calculation of first traveltimes in anisotropic media. *Geophysical Journal International* **113**, 318–342.

Mensch T. and Rasolofosaon P. 1997. Elastic-wave velocities in anisotropic media of arbitrary symmetry – generalization of Thomsen's parameters ε, δ and γ. *Geophysical Journal International* **128**, 43–64.

Podvin P. and Lecomte I. 1991. Finite difference computation of traveltimes in very contrasted velocity model: a massively parallel approach and its associated tools. *Geophysical Journal International* **105**, 271–284.

Qin F., Luo Y., Olsen K.B., Cai W. and Schuster G.T. 1992. Finite-difference solution of the eikonal equation along expanding wavefronts. *Geophysics* **57**, 478–487.

Van Trier J. and Symes W.W. 1991. Upwind finite-difference calculation of traveltimes. *Geophysics* **56**, 812–821.

Vidale J.E. 1988. Finite-difference calculation of traveltimes. *Bulletin of the Seismological Society of America* **78**, 2062–2076.

Vidale J.E. 1990. Finite-difference calculation of traveltimes in three dimensions. *Geophysics* **55**, 521–526.